

Application of Zacchaeus Integral Transform to Heat Conduction

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Abstract

This study propounded a novel integral transform called Zacchaeus transform. The functional characteristic of Zacchaeus transform is discussed. The Zacchaeus transform of elementary functions and some of the derivatives of a function are obtained. The initial value problem in heat conduction are analyzed by applying Zacchaeus transform. It is revealed that by applying Zacchaeus transform the initial value problems in heat conduction are easily analyzed.

Keywords: Zacchaeus transform, heat conduction, integral transform, initial condition.

Introduction

In literature, there are many integral transform such as: Laplace transform, Sumudu transform, Elzaki transform, Aboodh transform, Rohit transform, Mahgoub transform (laplace – carson transform), Mohand transform, Dinesh transform, Gupta transform, Kamal Transform, Sadik transform, Sawi transform, Tarig transform, Shehu transform, Natural transform, Z Z transform, G-transform, Pourresa transform, α - integral laplace transform, SEE transform and so on [3-10]. Many of them have been applied to solve differential equation. In this literature Zacchaeus transform will be applied to solve differential equation with initial value problem in heat conduction.

II. DEFINITION OF ZACCHAEUS INTEGRAL TRANSFORM

Let $f(t)$ be a continuous function on any interval for $t \geq 0$. The Zacchaeus transform of $f(t)$ is defined as $Z[f(t)] = V^4 \int_0^\infty e^{-vt} f(t) dt = z(v)$, provided that the integral convergence, where V may be a real or complex parameter and Z is the Zacchaeus transform operator.

III. ZACCHAEUS TRANSFORM OF ELEMENTARY FUNCTION

According to the definition of Zacchaeus transform $Z[f(t)] = V^4 \int_0^\infty e^{-vt} f(t) dt$

1. $Z[f(k)] = kv^3$

2a. $Z[f(t^n)] = \frac{n!v^3}{v^n}, n \geq 0$

2b. $Z[f(t^n)] = \frac{\Gamma(n+1)v^3}{v^n}, n > -1$

3. $Z[f(e^{kt})] = \frac{v^4}{v-k}, v > k$

4. $Z[f(\sin kt)] = \frac{kv^4}{v^2+k^2}$

5. $Z[f(\cos kt)] = \frac{v^5}{v^2+k^2}$

6. $Z[f(\sinh kt)] = \frac{kv^4}{v^2-k^2}$

7. $Z[f(\cosh kt)] = \frac{v^5}{v^2-k^2}$

Proof:

1. $Z(t) = v^4 \int_0^\infty e^{-vt} f(t) dt = v^4 \int_0^\infty ke^{-vt} dt = v^4 \cdot k \left[\frac{e^{-vt}}{-v} \right]_0^\infty = kv^3$

2. $Z(t^n) = v^4 \int_0^\infty e^{-vt} f(t) dt$ putting $vt = x$ then $t = \frac{x}{v} \Rightarrow dt = \frac{dx}{v}$,

Thus, we have $Z(t^n) = v^4 \int_0^\infty e^{-x} \left(\frac{x}{v}\right)^n \frac{dx}{v}$ or

$Z(t^n) = v^4 \cdot \frac{1}{v^{n+1}} \int_0^\infty e^{-x} x^n dx$

$Z(t^n) = \frac{n!v^4}{v^{n+1}} = \frac{n!v^3}{v^n} = \frac{\Gamma(n+1)v^3}{v^n}$

Since $\Gamma(n+1) = \int_0^\infty e^{-x} x^n dx$ and $\Gamma(n+1) = n!$

3. $Z(e^{kt}) = v^4 \int_0^\infty e^{-vt} \cdot e^{kt} dt = v^4 \int_0^\infty e^{-(v-k)t} dt = v^4 \left[\frac{e^{-(v-k)t}}{-(v-k)} \right] = \frac{v^4}{v-k}$

4. $Z(\sin kt) = Z \left[\frac{e^{-ikt} - e^{ikt}}{2i} \right] = \frac{1}{2i} \left[\frac{v^4}{v-ik} - \frac{v^4}{v+ik} \right] = \frac{kv^4}{v^2+k^2}$

5. $Z(\cos kt) = Z \left[\frac{e^{ikt} + e^{-ikt}}{2} \right] = \frac{1}{2} \left[\frac{v^4}{v-ik} - \frac{v^4}{v+ik} \right] = \frac{v^5}{v^2+k^2}$

6. $Z(\sinh kt) = Z \left[\frac{1}{2} (e^{kt} - e^{-kt}) \right] = \frac{1}{2} \left[\frac{v^4}{v-k} - \frac{v^4}{v+k} \right] = \frac{kv^4}{v^2-k^2}$

$$7. \quad Z[\cos h(kt)] = Z\left[\frac{e^{kt} + e^{-kt}}{2}\right] = \frac{1}{2}\left[\frac{v}{v-k} + \frac{v^4}{v+k}\right] = \frac{v^5}{v^2 - k^2}$$

IV. ZACCHAEUS TRANSFORM OF DERIVATIVES

- $Z[f^i(t)] = vz[f(t)] - v^4 f(0)$
- $Z[f^{ii}(t)] = v^2 z[f(t)] - v^5 f(0) - v^4 f^i(0)$
- $Z[f^{iii}(t)] = v^3 z[f(t)] - v^6 f(0) - v^5 f^i(0) - v^4 f^{ii}(0)$
- $Z[f^{iv}(t)] = v^4 z[f(t)] - v^7 f(0) - v^6 f^i(0) - v^5 f^{ii}(0) - v^4 f^{iii}(0)$
- $Z[f^n(t)] = v^n z[f(t)] - v^{(n+3)} f(0) - v^{(n+2)} f^i(0) - v^{(n+1)} f^{ii}(0) - \dots - v^n f^{(n-1)}(0)$

V. INVERSE ZACCHAEUS TRANSFORM OF ELEMENTARY FUNCTIONS

- $Z^{-1}(av^3) = a$
- $Z^{-1}(v^2) = t$
- $Z^{-1}(v) = \frac{t^2}{2!}$
- $Z^{-1}(v^{3-n}) = \frac{t^n}{n!}$
- $Z^{-1}(v^{3-n}) = \frac{t^n}{\Gamma(n+1)}$
- $Z^{-1}\left(\frac{v^4}{v-k}\right) = e^{kt}$
- $Z^{-1}\left(\frac{v^4}{v^2+k^2}\right) = \frac{\sin kt}{k}$
- $Z^{-1}\left(\frac{v^5}{v^2+k^2}\right) = \cos kt$
- $Z^{-1}\left(\frac{v^4}{v^2-k^2}\right) = \frac{\sin hkt}{k}$
- $Z^{-1}\left(\frac{v^5}{v^2-k^2}\right) = \cos hkt$

VI. APPLICATION OF ZACCHAEUS TRANSFORM

Application 1: The rate of decrease of temperature of the body is proportional to the difference between the temperature of the body and that of the medium.

We will apply Zacchaeus transform to find the temperature of the body at any instant.

Solution

Let T_1 be the initial temperature of the body, ' T ' be the temperature of the body at any instant t and T_0 be the temperature of the medium, then we have

$$T^1(t) = -k[T(t) - T_0]$$

Applying Zacchaeus transform, we have

$$vz(v) - v^4 z(0) = -kz(v) + v^3 kT_0 \quad \text{at} \quad t =$$

$0, T(0) = T_1$, therefore, we have

$$vz(v) - v^4 T_1 = -kz(v) + v^3 kT_0$$

$$z(v) = \frac{v^4 T_1}{v+k} + \frac{v^3 kT_0}{v+k}$$

$$z(v) = T_1 \left(\frac{v^4}{v+k}\right) + T_0 \left[v^3 - \frac{v^4}{v+k}\right]$$

Taking inverse Zacchaeus transform, we have

$$T(t) = T_1 e^{-kt} + T_0 [1 - e^{-kt}]$$

Application 2: The rate at which a body cool is proportional to the differences between the temperature of the body and that of the surrounding air. If a body in air at 25°C will cool from 100°C to 75°C in one minute, find its temperature at the end of three minutes.

Solution

Let temperature of the body be $T^\circ\text{C}$

$$\frac{dT}{dt} = k(T - 25) \text{ or } \frac{dT}{dt} = KT$$

Applying Zacchaeus integral transform

$$Z\left[\frac{dT}{dt}\right] = Z(KT)$$

$$VZ[T(t)] - v^4 T(0) = kZ[T(t)]$$

Introducing the initial condition

$$VZ[T(t)] - AV^4 = KZ[T(t)]$$

$$(V - K)Z[T(t)] = AV^4$$

$$Z[T(t)] = \frac{AV^4}{V-K}$$

Taking the inverse of Zacchaeus transform

$$T = Ae^{kt}$$

Since the temperature reduce by 25°C

$$T - 25 = Ae^{kt}$$

When $t = 0$, then $T = 100$, from $A = 75$

When $t = 1$, then $T = 75$ and $A = 75$ from

$$(2.1) \frac{2}{3} = e^k, \text{ then } k = \frac{8}{27}$$

$\therefore$ Therefore equation (2.1) becomes $T = 25 + 75e^{kt}$

When $t = 3$, the $T = 25 + 75e^{3k}$

$$= 25 + 75 \times (e^k)^3 = 25 + 75 \times \left(\frac{2}{3}\right)^3$$

$$= 25 + 75 \times \frac{8}{27} = 25 + \frac{600}{27}$$

$$= 25 + 22.2222 \approx 47.22^\circ\text{C}$$

Application 3: The rate at which the ice melts is proportional to the amount of ice present at

the instant. Find the amount of ice left after 2hours if half of the quantity melt in 30minutes

Solution

Let M be the amount of ice at any time t

$$\frac{dm}{dt} = km \text{ or } \frac{dm}{dm} = kdt \dots\dots\dots (3.1)$$

Applying Zacchaeus transform

$$Z\left[\frac{dm}{dt}\right] = Z(km)$$

$$VZ(m) - v^4 M(0) = KZ(m)$$

Introducing the initial condition At $t = 0, m = M$

$$VZ(m) - MV^4 = KZ(m)$$

$$(V - K)Z(m) = MV^4$$

$$Z(m) = \frac{MV^4}{V-K}$$

Taking the inverse Zacchaeus transform

$$m = Me^{kt} \dots\dots\dots (3.2)$$



At $t = 30 \text{ minutes} = \frac{1}{2} \text{ hours}$

$$m = Me^{kt} = Me^{\frac{k}{2}}$$

$$m = \frac{M}{2} \text{ when } t = \frac{1}{2} \text{ hour}$$

$$\frac{M}{2} = Me^{\frac{k}{2}}$$

$$M = 2Me^{\frac{k}{2}}$$

$$1 = 2e^{\frac{k}{2}} = e^{\frac{k}{2}} = \frac{1}{2}$$

Then $K = 2\log\frac{1}{2}$ or 1.3863 on putting the value of k in 3.2, we have

$$\log m = \left(2\log\frac{1}{2}\right)t + \log M \dots\dots\dots (3.3)$$

on putting $t = 2 \text{ hours}$ in 3, we have $\log m = 4\log\frac{1}{2} + \log M$

$$= \log \frac{m}{M} = \log \left(\frac{1}{2}\right)^4 \text{ or } \frac{m}{M} = \frac{1}{16} \text{ or } \frac{M}{16}$$

After 2 hours, amount of ice left $= \frac{1}{16}$ of the amount of ice at the beginning.

Application 4: Under certain conditions, cane sugar is converted into dextrone at a rate, which is proportional to the amount unconverted at any time. If out of 75grams of sugar at $t = 0$, 8grams are converted during the first 3minutes, find the amount converted on $\frac{1}{2}$ hours

Solution

Let A_0 be the amount of cane sugar initially, A_c be the amount of cane sugar converted into dextrone. Then according to problem,

$$\frac{dA_c}{dt} = k(A_0 - A_c) \quad \text{or} \quad \frac{dA_c}{dt} = kA_0 - kA_c \dots\dots\dots (4.0)$$

$$\frac{dA_c}{dt} + kA_c = kA_0 \dots (4.1)$$

Which is a linear differential equation Taken zaccchaus transform of (4.1)

$$Z\left[\frac{dA_c}{dt}\right] + kZ(A_c) = kZ(A_0)$$

$$VZ(A_c) - V^4A_c(0) + kZ(A_c) = kZ(A_0)$$

$$(V + K)Z(A_c) = V^4A_c(0) + kZ(A_0)$$

$$Z(A_c) = \frac{V^4A_c(0)}{V+K} + \frac{kZ(A_0)}{V+K}$$

Taking the inverse zaccchaus transform

$$A_c = A_c(0)e^{-kt} + A_0 \dots (4.2)$$

$$\text{At } t = 0, A_c = 0 \text{ and } A_0 = 75$$

$$0 = A_c(0)e^{-kt} + 75$$

$$0 = A_c(0)e^{-k(0)} + 75 \dots (4.3)$$

$$-75 = A_c(0)$$

$$\text{At } t = 30, A_c = 8$$

$$8 = 75 - 75e^{-30k}$$

$$8 - 75 = -75e^{-30k}$$

$$e^{-30k} = \frac{67}{75} \dots (4.4)$$

At $t = 90$, (4.3) becomes

$$A_c = 75 - 75e^{-90k}$$

$$A_c = 75 - 75\left(\frac{67}{75}\right)^3 \text{ from (4.4)}$$

$$A_c = 75 - \frac{(67)^3}{(75)^2} = 75 - \frac{300763}{5625}$$

$$A_c = 75 - 53.45 = 21.55$$

Conclusion

In this paper, we have applied a modified version of Laplace transform, namely Zaccchaus transform for solving problems in heat conduction. Like other transform, Zaccchaus transform is discovered to be a very effective method for solving initial value problems in heat condition.

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